Face Alignment Through 2.5D Active Appearance Models

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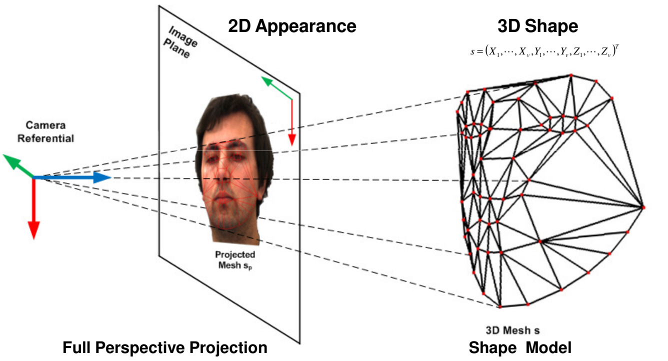
Abstract

- 3D face models from single view through 2.5D Active Appeance Models.
- The 2.5D AAM combine a 3D Point Distribution Model (PDM) and 2D appearance whose control points are defined by full perspective projections of the PDM.
- Two fitting algorithms - Simultaneous Forwards Additive (SFA) and Normalization Forwards Additive (NFA) - and their computationally efficient approximations are proposed, based on the Lucas and Kanade framework. Expanded solutions for the SFA and NFA are also proposed taking into account head self occlusions.

2.5D Parametric Models



$s = (X_1, \cdots, X_v, Y_1, \cdots, Y_v, Z_1, \cdots, Z_v)^T$

Full Perspective Projection

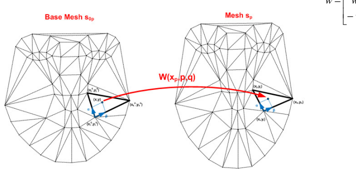
$$\begin{bmatrix} w(x_1 \cdots x_v) \\ w(y_1 \cdots y_v) \\ w \cdots w \end{bmatrix} = \underbrace{\begin{bmatrix} f_x & \alpha_s & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{K} \underbrace{[R_0 \quad t_0]}_{BasePose} \begin{bmatrix} s^{x_1} \cdots s^{x_v} \\ s^{y_1} \cdots s^{y_v} \\ s^{z_1} \cdots s^{z_v} \\ 1 \cdots 1 \end{bmatrix}$$

Shape Model

$$s = s_0 + \sum_{i=1}^{n} p_i \phi_i + \sum_{j=1}^{6} q_j \psi_j + \underbrace{\int_0^{t-1} \sum_{j=1}^{6} q_j \psi_j \partial t}_{s_\psi}$$

p – shape parameters
q – pose parameters

Piecewise Affine Warp



$$W(x_p, p, q) = x_{p_i} + \alpha(x_{p_j} - x_{p_i}) + \beta(x_{p_k} - x_{p_i}), \forall triangles \in s_{0_p}$$

$$w = \begin{bmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{bmatrix}$$

$$R(w, \theta) = e^{w\theta} = \sum_{n=0}^{\infty} \frac{(w\theta)^n}{n!}$$

$$R(w, \theta) = I_3 + w\theta + \frac{(w\theta)^2}{2!} + \frac{(w\theta)^3}{3!} + \cdots$$

$$R(w, \theta) = I_3 + w\sin(\theta) + w^2(1 - \cos(\theta))$$

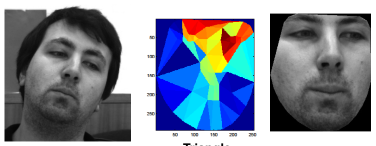
$$R(w, \theta) \approx I_3 + w\theta$$

Representing Pose Variation

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} 1 & -w_z & w_y \\ w_z & 1 & -w_x \\ -w_y & w_x & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} + \begin{bmatrix} 0 & Z & -Y & 1 & 0 & 0 \\ -Z & 0 & X & 0 & 1 & 0 \\ Y & -X & 0 & 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} w_x \\ w_y \\ w_z \\ t_x \\ t_y \\ t_z \end{bmatrix}}_{q}$$

$$\Psi = \underbrace{\begin{bmatrix} 0 & s_0^{z_1} & -s_0^{y_1} & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & s_0^{z_v} & -s_0^{y_v} & 1 & 0 & 0 \\ -s_0^{z_1} & 0 & s_0^{x_1} & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ -s_0^{z_v} & 0 & s_0^{x_v} & 0 & 1 & 0 \\ s_0^{y_1} & -s_0^{x_1} & 0 & 0 & 0 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ s_0^{y_v} & -s_0^{x_v} & 0 & 0 & 0 & 1 \end{bmatrix}}_{\psi_1 \cdots \psi_6}$$



Appearance Model

$$A(x_p) = A_0(x_p) + \sum_{i=1}^{m+2} \lambda_i A_i(x_p), x_p \in s_{0_p}$$

Model Fitting

$$\sum_{x_p \in s_{0_p}} \left[A_0(x_p) + \sum_{i=1}^{m+2} \lambda_i A_i(x_p) - I(W(x_p, p, q)) \right]^2 \Leftrightarrow \arg\min_{p,q,\lambda} \sum_{x_p \in s_{0_p}}$$

Simultaneous Forwards Additive (SFA)

Warp Input Image and the gradients
$I(W(x_p, p, q)) \qquad \nabla I(W(x_p, p, q))$

Compute the Error Image
$E(x_p) = A_0(x_p) + \sum_{i=1}^{m+2} \lambda_i A_i(x_p) - I(W(x_p, p, q))$

Find the Jacobian of the Warp w.r.t Shape and Pose
$\frac{\partial W(x_p, p, q)}{\partial p} \qquad \frac{\partial W(x_p, p, q)}{\partial p}$

Compute the Steepest Descent images
$SD(x_p)_{sfa} = \left[\nabla I \frac{\partial W(x_p,p,q)}{\partial p_1} \cdots \nabla I \frac{\partial W(x_p,p,q)}{\partial p_n} \; \nabla I \frac{\partial W(x_p,p,q)}{\partial q_1} \cdots \nabla I \frac{\partial W(x_p,p,q)}{\partial q_6} \; A_1(x_p) \cdots A_{m+2}(x_p) \right]$

Find the Hessian and its inverse
$H_{sfa} = \sum_{x_p \in s_{0_p}} SD(x_p)^T_{sfa} SD(x_p)_{sfa}$

Compute parameters update
$\begin{bmatrix} \Delta p \\ \Delta q \\ \Delta\lambda \end{bmatrix} = H^{-1}_{sfa} \sum_{x_p \in s_{0_p}} SD(x_p)^T_{sfa} E(x_p)_{sfa} \qquad \begin{bmatrix} p \\ q \\ \lambda \end{bmatrix} \leftarrow \begin{bmatrix} p \\ q \\ \lambda \end{bmatrix} + \begin{bmatrix} \Delta p \\ \Delta q \\ \Delta\lambda \end{bmatrix}$

Until |Δp, Δq, Δλ|<ε or maximum iterations

Normalization Forwards Additive (NFA)

Warp Input Image and the gradients
$I(W(x_p, p, q)) \qquad \nabla I(W(x_p, p, q))$

Compute the Error Image
$E(x_p)_{lk} = A_0(x_p) - I(W(x_p, p, q))$

Project the Error Image into Ai(x) and Normalize
$\lambda = \sum_{i=1}^{m+2} A_i(x_p) E(x_p)_{lk} \qquad E(x_p)_{nfa} = E(x_p)_{lk} - \sum_{i=1}^{m+2} \lambda_i A_i(x_p)$

Find the Jacobian of the Warp w.r.t Shape and Pose
$\frac{\partial W(x_p, p, q)}{\partial p} \qquad \frac{\partial W(x_p, p, q)}{\partial p}$

Compute the Steepest Descent images
$SD(x_p)_{nfa} = \left[\nabla I \frac{\partial W(x_p,p,q)}{\partial p_1} \cdots \nabla I \frac{\partial W(x_p,p,q)}{\partial p_n} \; \nabla I \frac{\partial W(x_p,p,q)}{\partial q_1} \cdots \nabla I \frac{\partial W(x_p,p,q)}{\partial q_6} \right]$

Find the Hessian and its inverse
$H_{nfa} = \sum_{x_p \in s_{0_p}} SD(x_p)^T_{nfa} SD(x_p)_{nfa}$

Compute parameters update
$\begin{bmatrix} \Delta p \\ \Delta q \end{bmatrix} = H^{-1}_{nfa} \sum_{x_p \in s_{0_p}} SD(x_p)^T_{nfa} E(x_p)_{nfa} \qquad \begin{bmatrix} p \\ q \end{bmatrix} \leftarrow \begin{bmatrix} p \\ q \end{bmatrix} + \begin{bmatrix} \Delta p \\ \Delta q \end{bmatrix}$

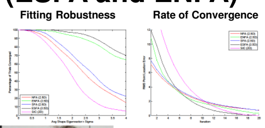
Until |Δp, Δq|<ε or maximum iterations

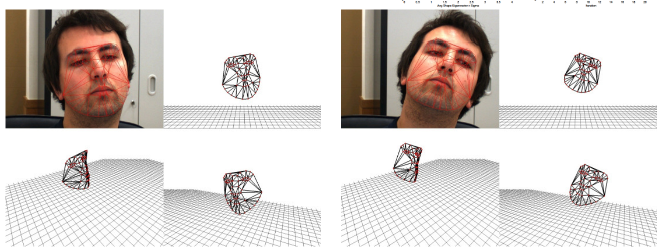
Efficient Approximations (ESFA and ENFA)

$$\left(A_0(x_p) + \sum_{i=1}^{m+2} \lambda_i A_i(x_p) \right) \approx I(W(x_p, p, q)) \Rightarrow \underbrace{\left(\nabla A_0(x_p) + \sum_{i=1}^{m+2} \lambda_i \nabla A_i(x_p) \right)}_{\nabla A(x_p, \lambda)} \approx \nabla I(W(x_p, p, q))$$

$$SD(x_p)_{esfa} = \left[\nabla A(x_p,\lambda) \frac{\partial W(x_p,p,q)}{\partial p_1} \cdots \nabla A(x_p,\lambda) \frac{\partial W(x_p,p,q)}{\partial p_n} \; \nabla A(x_p,\lambda) \frac{\partial W(x_p,p,q)}{\partial q_1} \cdots \nabla A(x_p,\lambda) \frac{\partial W(x_p,p,q)}{\partial q_6} \; A_1(x_p) \cdots A_{m+2}(x_p) \right]$$

$$SD(x_p)_{enfa} = \left[\nabla A(x_p,\lambda) \frac{\partial W(x_p,p,q)}{\partial p_1} \cdots \nabla A(x_p,\lambda) \frac{\partial W(x_p,p,q)}{\partial p_n} \; \nabla A(x_p,\lambda) \frac{\partial W(x_p,p,q)}{\partial q_1} \cdots \nabla A(x_p,\lambda) \frac{\partial W(x_p,p,q)}{\partial q_6} \right]$$





Robust Fitting (RSFA,RNFA,ERSFA,ERNFA)

$$\sum_{x_p \in s_{0_p}} \rho\left(E(x_p)^2_{sfa}, \sigma\right) \Leftrightarrow \arg\min_{p,q,\lambda} \sum_{x_p \in s_{0_p}}$$

RSFA Parameters Updates (IRLS)

$$\begin{bmatrix} \Delta p \\ \Delta q \\ \Delta\lambda \end{bmatrix} = H^{-1}_{rsfa} \sum_{x_p \in s_{0_p}} \rho'(E(x_p)^2_{sfa}) SD(x_p)^T_{sfa} E(x_p)_{sfa}$$

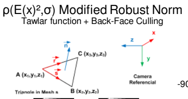
$$H_{rsfa} = \sum_{x_p \in s_{0_p}} \rho'(E(x_p)^2_{sfa}) SD(x_p)^T_{sfa} SD(x_p)_{sfa}$$

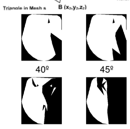
RNFA Parameters Updates (IRLS)

$$\begin{bmatrix} \Delta p \\ \Delta q \end{bmatrix} = H^{-1}_{rnfa} \sum_{x_p \in s_{0_p}} \rho'(E(x_p)^2_{nfa}) SD(x_p)^T_{nfa} E(x_p)_{nfa}$$

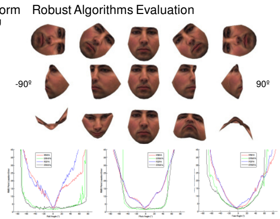
$$H_{rnfa} = \sum_{x_p \in s_{0_p}} \rho'(E(x_p)^2_{nfa}) SD(x_p)^T_{nfa} SD(x_p)_{nfa}$$

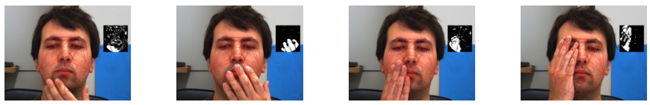
ρ(E(x)²,σ) Modified Robust Norm
Tawlar function + Back-Face Culling





Robust Algorithms Evaluation





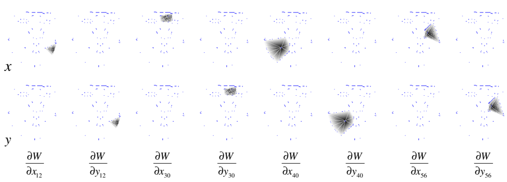
The Jacobian of the Warp

$$\frac{\partial W(x_p, p, q)}{\partial x_k} = (1 - \alpha - \beta, 0)$$

$$\frac{\partial W(x_p, p, q)}{\partial y_k} = (0, 1 - \alpha - \beta)$$

α,β are barycentric coordinates

(Sparse Matrices)



$\frac{\partial W}{\partial x_{12}} \quad \frac{\partial W}{\partial y_{12}} \quad \frac{\partial W}{\partial x_{30}} \quad \frac{\partial W}{\partial y_{30}} \quad \frac{\partial W}{\partial x_{40}} \quad \frac{\partial W}{\partial y_{40}} \quad \frac{\partial W}{\partial x_{56}} \quad \frac{\partial W}{\partial y_{56}}$

Shape

$$\frac{\partial W(x_p, p, q)}{\partial p} = \sum_{k=1}^{v} \left(\frac{\partial W(x_p, p, q)}{\partial x_k} \frac{\partial x_k}{\partial p} + \frac{\partial W(x_p, p, q)}{\partial y_k} \frac{\partial y_k}{\partial p} \right)$$

$$\begin{bmatrix} wx_k \\ wy_k \\ w \end{bmatrix} = K[R_0 \quad t_0] \begin{bmatrix} s_0^{x_k} + p_i \phi_i^{x_k} \\ s_0^{y_k} + p_i \phi_i^{y_k} \\ s_0^{z_k} + p_i \phi_i^{z_k} \\ 1 \end{bmatrix}_{\substack{i=1 \cdots n \\ k=1 \cdots v}} \qquad \frac{\partial x_k}{\partial p} = \frac{\partial}{\partial p}\left(\frac{wx_k}{w} \right) \quad \frac{\partial y_k}{\partial p} = \frac{\partial}{\partial p}\left(\frac{wy_k}{w} \right)$$

(scalars)

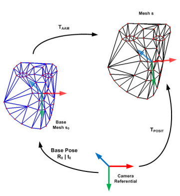
Pose

$$\frac{\partial W(x_p, p, q)}{\partial q} = \sum_{k=1}^{v} \left(\frac{\partial W(x_p, p, q)}{\partial x_k} \frac{\partial x_k}{\partial q} + \frac{\partial W(x_p, p, q)}{\partial y_k} \frac{\partial y_k}{\partial q} \right)$$

$$\begin{bmatrix} wx_k \\ wy_k \\ w \end{bmatrix} = K[R_0 \quad t_0] \begin{bmatrix} s_0^{x_k} + q_j \psi_j^{x_k} + s_\psi^{x_k} \\ s_0^{y_k} + q_j \psi_j^{y_k} + s_\psi^{y_k} \\ s_0^{z_k} + q_j \psi_j^{z_k} + s_\psi^{z_k} \\ 1 \end{bmatrix}_{\substack{j=1 \cdots 6 \\ k=1 \cdots v}} \qquad \frac{\partial x_k}{\partial q} = \frac{\partial}{\partial q}\left(\frac{wx_k}{w} \right) \quad \frac{\partial y_k}{\partial q} = \frac{\partial}{\partial q}\left(\frac{wy_k}{w} \right)$$

(scalars)

Initial Estimate



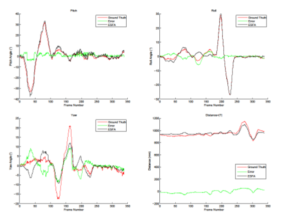
The initial 6DOF combines:
face detection (Adaboost)
+
(POSIT) Pose from Orthography and Scaling with Iterations

using s0 as a rigid model.

$$T_{AAM} = \begin{bmatrix} R_0 & t_0 \\ 0 & 1 \end{bmatrix}^{-1} T_{POSIT}$$

Head Pose Estimation

Analysing pose updates Δq (ESFA Algorithm)



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